Abstract:
The proximal point method is a fundamental conceptual algorithm proposed by Martinet (1972) and Rockafellar (1976) for solving monotone inclusions (the set-valued analog of equation solving which includes, as special cases, many classical PDEs). We consider a framework of inexact proximal point (IPP) methods for convex optimization (CO) which allows a relative error tolerance in the approximate solution of each proximal subproblem. This framework, which we refer to as the IPP-CO framework, is a subset of the hybrid proximal extragradient (HPE) method introduced by Solodov and Svaiter for solving monotone inclusion problems. Global convergence rate results for the HPE method have been derived previously, and hence apply to the IP P-CO framework. However, by exploiting the special structure of convex optimization, convergence rate results stronger than those obtained for the HPE method are derived for the IPP-CO framework.We show, as illustration, that the well-known forward-backward splitting method for convex optimization belongs to the IPP-CO framework and, as a consequence, we derive iteration-complexity bounds similar to, but under more general assumptions than have appeared in the literature. More specifically, the state-of-the-art assumes that the sublevel subsets of the objective function are bounded and express the complexity bounds in terms of the diameter of the sublevel set corresponding to the initial iterate. On the other hand, our results do not assume boundedness of the sublevel sets and express the bounds in items of the distance of the initial iterate to the optimal solution set.
We also consider convex optimization problems whose objective functions are obtained by maxi-mizing convex-concave saddle functions and propose an inexact forward-backward splitting algorithm for solving them. The inexactness of the proposed method originates from the assumption that the objective function and its gradient are approximately evaluated in the sense that the corresponding saddle function maximization subproblem is solved inexactly. Iteration-complexity bounds are ob-tained for the inexact forward-backward splitting algorithm by showing that it also belongs to the IPP-CO framework.